Appendix: Uncharted but not uninfluenced: Influence maximization with an uncertain network

A Proofs

Here we prove technical lemmas which were deferred from the main text. All claims are (where applicable) proved for adaptive policies; the single stage case follows by simply restricting the argument to 1-stage policies. Analogously to the definitions given in Section 5, $g_\pi(\boldsymbol{\theta})$ gives the expected influence spread of a policy π in Lemma 1 for any prior vector $\boldsymbol{\theta}$. g_G gives the expected influence spread of the dynamic greedy algorithm (or any other benchmark algorithm).

Lemma 1 (restated for policies): *For any policy π and any $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2 \in \mathcal{P}$, $|g_\pi(\boldsymbol{\theta}_1) - g_\pi(\boldsymbol{\theta}_2)| \leq nT||\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2||_1$. The same holds for g_G.*

Proof. Each edge e with type i draws a propagation probability $p_e \sim \theta_e$. Equivalently, we can view each edge as independently drawing the number of steps t_e until it will activate. In our case, t_e follows a geometric distribution with success probability p_e. We can write the influence spread of any policy π as the expectation over the number of nodes which are reached under any fixed setting of the random variables t_e. Let $\boldsymbol{t}$ be the vector containing t_e for each $e \in E$. Then define $\sigma(\pi, \boldsymbol{t})$ to be the expected number of nodes which are reachable from π's selections in at most T steps given the transmission times. This expectation is only over any randomness introduced by π itself; for a fixed set of seed nodes, σ is deterministic. We can write

$$\begin{aligned} g_\pi(\boldsymbol{\theta}) &= \sum_{t_{e_1}=1}^{\infty} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} Pr(\boldsymbol{t}|\boldsymbol{\theta})\sigma(\pi, \boldsymbol{t}) \\ &= \sum_{t_{e_1}=1}^{\infty} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t}) \end{aligned} \tag{1}$$

Now we take the derivative with respect to θ_e for a fixed e. Without loss of generality, take θ_{e_1}:

$$\begin{aligned}\frac{\partial g_\pi(\theta)}{\partial \theta_{e_1}} &= \frac{\partial}{\partial \theta_{e_1}}\left[\sum_{t_{e_1}=1}^{\infty}\sum_{t_{e_2}=1}^{\infty}\cdots\sum_{t_{e_m}=1}^{\infty}\prod_{e\in E} Pr(t_e|\theta_e)\sigma(\pi,\mathbf{t})\right] \\ &= \sum_{t_{e_1}=1}^{\infty}\sum_{t_{e_2}=1}^{\infty}\cdots\sum_{t_{e_m}=1}^{\infty}\frac{\partial}{\partial \theta_{e_1}}\left[\prod_{e\in E} Pr(t_e|\theta_e)\right]\sigma(\pi,\mathbf{t}) \\ &= \sum_{t_{e_1}=1}^{\infty}\sum_{t_{e_2}=1}^{\infty}\cdots\sum_{t_{e_m}=1}^{\infty}\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right]\prod_{e\in E\setminus\{e_1\}} Pr(t_e|\theta_e)\sigma(\pi,\boldsymbol{t})\end{aligned}$$

To bound this sum, we first investigate $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right]$. Since e_1 attempts to activate each step, for a fixed success probability p_{e_1}, t_{e_1} follows a geometric distribution supported on $[1,\infty)$. We assume that the success probability p_e follows a uniform distribution with center θ_{e_1} and a fixed width w. Hence, we obtain

$$Pr(t_{e_1}|\theta_{e_1}) = \frac{1}{w}\int_{p_{e_1}=\theta_{e_1}-w/2}^{\theta_{e_1}+w/2}(1-p_{e_1})^{t_{e_1}-1}p_{e_1}\,dp_{e_1}.$$

Differentiating with respect to θ_{e_1}, we have

$$\begin{aligned}\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] &= \frac{1}{w}\frac{\partial}{\partial \theta_{e_1}}\int_{p_{e_1}=\theta_{e_1}-w/2}^{\theta_{e_1}+w/2}(1-p_{e_1})^{t_{e_1}-1}p_{e_1}\,dp_{e_1} \\ &= \frac{1}{w}\left[\left(1-\left(\theta_{e_1}+\frac{w}{2}\right)\right)^{t_{e_1}-1}\left(\theta_{e_1}+\frac{w}{2}\right)-\left(1-\left(\theta_{e_1}-\frac{w}{2}\right)\right)^{t_{e_1}-1}\left(\theta_{e_1}-\frac{w}{2}\right)\right]\end{aligned}$$

Next, we establish three useful properties of $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right]$.

Claim 1: For any value of t_{e_1}, $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \le 1$. The proof is by induction on t_{e_1}. Starting with $t_{e_1}=1$, we have

$$\begin{aligned}\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}=1|\theta_{e_1})\right] &= \frac{1}{w}\left[\left(\theta_{e_1}+\frac{w}{2}\right)-\left(\theta_{e_1}-\frac{w}{2}\right)\right] \\ &= 1\end{aligned}$$

Next, we show that for any t, $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t+1|\theta_{e_1})\right] \le \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t|\theta_{e_1})\right]$. Expanding $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t+1|\theta_{e_1})\right]$ gives

$$\begin{aligned}\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t+1|\theta_{e_1})\right] &= \frac{1}{w}\left[\left(1-\left(\theta_{e_1}+\frac{w}{2}\right)\right)^{t}\left(\theta_{e_1}+\frac{w}{2}\right)-\left(1-\left(\theta_{e_1}-\frac{w}{2}\right)\right)^{t}\left(\theta_{e_1}-\frac{w}{2}\right)\right] \\ &= \frac{1}{w}\left[\left(1-\left(\theta_{e_1}+\frac{w}{2}\right)\right)\left(1-\left(\theta_{e_1}+\frac{w}{2}\right)\right)^{t-1}\left(\theta_{e_1}+\frac{w}{2}\right)\right]- \\ &\qquad \frac{1}{w}\left[\left(1-\left(\theta_{e_1}-\frac{w}{2}\right)\right)\left(1-\left(\theta_{e_1}-\frac{w}{2}\right)\right)^{t-1}\left(\theta_{e_1}-\frac{w}{2}\right)\right]\end{aligned}$$

Since $1-(\theta_{e_1}+\frac{w}{2}) \le 1-(\theta_{e_1}-\frac{w}{2})$, this implies that $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t+1|\theta_{e_1})\right] \le \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t|\theta_{e_1})\right]$. Therefore, the claim holds for all t by induction.

Claim 2: $\sum_{t_{e_1}=1}^{\infty}\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right]=0$. This holds since $\sum_{t_{e_1}}^{\infty} Pr(t_{e_1}|\theta_{e_1})=1$.

Claim 3: Let $T' = \min\{t \,|\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t|\theta_{e_1})\right] \leq 0\}$ (T' must exist by Claim 2). Then, for any $t \geq T'$, $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t|\theta_{e_1})\right] \leq 0$. The proof is immediate using $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t+1|\theta_{e_1})\right] \leq \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t|\theta_{e_1})\right]$ from the proof of Claim 1.

Next, we deal with the inner term $\prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})$. Specifically, we need to establish how $\sigma(\pi, \boldsymbol{t})$ varies with t_{e_1}. For $t_{e_1} \leq T$, the exact value of t_{e_1} could change $\sigma(\pi, \boldsymbol{t})$. However, once $t_{e_1} > T$, influence will never spread fast enough along the edge to impact the objective (since we only count nodes influenced before the time horizon). We consider two cases.

Case 1: $T' > T$.

This allows us to split up the summation in $\frac{\partial g_\pi(\theta)}{\partial \theta_{e_1}}$ as follows:

$$\begin{aligned}
\frac{\partial g_\pi(\theta)}{\partial \theta_{e_1}} &= \sum_{t_{e_1}=1}^{T} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t}) + \qquad (2)\\
&\quad \sum_{t_{e_1}=T+1}^{\infty} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})\\
&\leq n \sum_{t_{e_1}=1}^{T} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e) + \qquad (3)\\
&\quad \sum_{t_{e_1}=T+1}^{\infty} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})\\
&= n \sum_{t_{e_1}=1}^{T} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e) + \qquad (4)\\
&\quad \sum_{t_{e_1}=T+1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})\\
&\leq nT + \left[\sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})\right] \sum_{t_{e_1}=T+1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \qquad (5)\\
&\leq nT + \left[\sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})\right] \sum_{t_{e_1}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right] \qquad (6)\\
&= nT \qquad (7)
\end{aligned}$$

(3) holds because each term in the first summation is positive since $T' > T$, and σ is never more than n. (4) holds from the fact that in both summations, the rest of the summand is now independent of $\frac{\partial}{\partial \theta_{e_1}}\left[Pr(t_{e_1}|\theta_{e_1})\right]$, which allows us to factor it out. (5) holds from Claim 1 and the fact that $\sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e) = 1$, as well as the fact that in the second summation, the summations over $t_{e_2}...t_{e_n}$ are now independent of t_{e_1}. (4) holds because of the definition of T', which ensure that all of the extra terms we add are positive. (5) holds because of Claim 2.

Case 2: $T' \leq T$.

In this case, we split the summation at T':

$$\frac{\partial g_\pi(\theta)}{\partial \theta_{e_1}} = \sum_{t_{e_1}=1}^{T'} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t}) + \tag{8}$$

$$\sum_{t_{e_1}=T'+1}^{\infty} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t})$$

$$\leq \sum_{t_{e_1}=1}^{T'} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e)\sigma(\pi, \boldsymbol{t}) \tag{9}$$

$$\leq n \sum_{t_{e_1}=1}^{T'} \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})] \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e) \tag{10}$$

$$= n \sum_{t_{e_1}=1}^{T'} \frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})] \sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e) \tag{11}$$

$$\leq n \sum_{t_{e_1}=1}^{T'} \frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})] \tag{12}$$

$$\leq nT' \tag{13}$$

$$\leq nT \tag{14}$$

(9) holds because the second summation in (8) must be negative by definition of T' combined with Claim 3. (10) holds because each term in the remaining summation is positive and σ is never more than n. (11) holds since we can now factor out $\frac{\partial}{\partial \theta_{e_1}} [Pr(t_{e_1}|\theta_{e_1})]$. (12) holds because $\sum_{t_{e_2}=1}^{\infty} ... \sum_{t_{e_m}=1}^{\infty} \prod_{e \in E \setminus \{e_1\}} Pr(t_e|\theta_e) = 1$. (13) holds by Claim 1. (14) holds because $T' \leq T$.

In both cases, we have that $||\nabla g_\pi(\boldsymbol{\theta})||_\infty \leq nT$, which implies that $|g_\pi(\theta_1) - g_\pi(\theta_2)| \leq nT||\theta_1 - \theta_2||_1$.

Now we extend this reasoning to g_G. Fix some $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ and note that we can partition the line segment connecting $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ into intervals where the policy selected by greedy does not change. Inspecting Equation 1, we know that the marginal gain $\Delta(u|\psi)$ to picking a node u at a partial realization ψ is a polynomial in $\boldsymbol{\theta}$ and hence continuous. So, for any $\boldsymbol{\theta}$ and any ψ, there must be an ℓ_1 ball $B_{\boldsymbol{\theta}}^{\psi}$ centered on $\boldsymbol{\theta}$ such that $u = \arg\max \Delta_{\boldsymbol{\theta}'}(u|\psi)\ \forall \boldsymbol{\theta}' \in B_{\boldsymbol{\theta}}^{\psi}$. That is, the maximal u does not change across $B_{\boldsymbol{\theta}}^{\psi}$. Since there a finite number of possible ψ, we can take the intersection to define a ball $B_{\boldsymbol{\theta}} = \bigcap_{\psi} B_{\boldsymbol{\theta}}^{\psi}$. We know that greedy will output the same policy for any point in this ball; that is, there is a policy π such that $g_G(\boldsymbol{\theta}') = g_\pi(\boldsymbol{\theta}')$ for any $\boldsymbol{\theta}' \in B_{\boldsymbol{\theta}}$. Let r be the furthest point on the $\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2$ line segment which still lies in $B_{\boldsymbol{\theta}}$. By applying our earlier conclusion for fixed policies to g_π, we know that $|g_G(\boldsymbol{\theta}_1) - g_G(r)| \leq nm||\boldsymbol{\theta}_1 - r||_1$. By iteratively applying the same argument to r until we arrive at a ball containing $\boldsymbol{\theta}_2$, we obtain that $|g_G(\boldsymbol{\theta}_1) - g_G(\boldsymbol{\theta}_2)| \leq nT||\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2||_1$. □

Lemma 1 allows us to easily prove Lemma 2, establishing the existence of a suitable discretization $\mathcal{P}^*$.

Lemma 2: (restated for policies) *Fix $\epsilon > 0$, and construct $\mathcal{P}^*$ using a grid with $\left(\frac{2nmT}{\epsilon}\right)^{|\Theta|}$ points. Then for any policy π and any point $\boldsymbol{\theta} \in \mathcal{P}$ there is a $\boldsymbol{\theta}^* \in \mathcal{P}^*$ satisfying $|R_G(\pi, \boldsymbol{\theta}) - R_G(\pi, \boldsymbol{\theta}^*)| \leq \epsilon$.*

Proof of Lemma 2. Recall the the set of allowed values for each $\theta \in \Theta$ is the hyperrectable $\times_\theta[a_\theta, b_\theta]$. Each point in this hyperrectangle maps to a point in $\mathcal{P}$ via duplicating each value θ across all of the edges that have $\theta_e = \theta$. Consider a grid over $\times_\theta[a_\theta, b_\theta]$ with $\left(\frac{2nmT}{\epsilon}\right)^{|\Theta|}$ points. We will let $\mathcal{P}^*$ be the set of $\boldsymbol{\theta}$

vectors corresponding to each point on this grid. Note that for any $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$ which are neighbors on the grid, $||\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2||_1 \leq \frac{\epsilon}{2nT}$ since the mapping from $\times_\theta[a_\theta, b_\theta]$ to $\mathcal{P}$ increases the ℓ_1 distance between two points by at most a factor m. Therefore, for *any* $\boldsymbol{\theta} \in \mathcal{P}$, there is a $\boldsymbol{\theta}^* \in \mathcal{P}^*$ which satisfies $||\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2||_1 \leq \frac{\epsilon}{2nT}$. By Lemma 1 we have $g_G(\boldsymbol{\theta_2}) \leq (1 + \frac{\epsilon}{2})g_G(\boldsymbol{\theta_1})$ and $g_\pi(\boldsymbol{\theta}_1) \leq (1 + \frac{\epsilon}{2})g_\pi(\boldsymbol{\theta}_2)$ (since $g_\pi(\boldsymbol{\theta}) \geq 1$ for any $\boldsymbol{\theta}, \pi$). Therefore, we have

$$\begin{aligned}
R(\boldsymbol{\theta}, \pi) - R(\boldsymbol{\theta}^*, \pi) &= \frac{g_\pi(\boldsymbol{\theta})}{g_G(\boldsymbol{\theta})} - \frac{g_\pi(\boldsymbol{\theta}^*)}{g_G(\boldsymbol{\theta}^*)} \\
&\leq \frac{g_\pi(\boldsymbol{\theta})}{g_G(\boldsymbol{\theta})} - \frac{g_\pi(\boldsymbol{\theta}^*)}{(1 + \frac{\epsilon}{2})g_G(\boldsymbol{\theta})} \\
&= \frac{(1 + \frac{\epsilon}{2})g_\pi(\boldsymbol{\theta}) - g_\pi(\boldsymbol{\theta}^*)}{(1 + \frac{\epsilon}{2})g_G(\boldsymbol{\theta})} \\
&\leq \frac{\frac{\epsilon}{2} + \frac{\epsilon}{2}g_\pi(\boldsymbol{\theta})}{(1 + \frac{\epsilon}{2})g_G(\boldsymbol{\theta})} \quad \text{(by Lemma 1)} \\
&\leq \frac{\frac{\epsilon}{2} + \frac{\epsilon}{2}g_G(\boldsymbol{\theta})}{(1 + \frac{\epsilon}{2})g_G(\boldsymbol{\theta})} \\
&\leq \epsilon.
\end{aligned}$$

This establishes that for any point in $\mathcal{P}$, there is a point in $\mathcal{P}^*$ with value within an additive ϵ. □

Next, we prove Lemma 3, that greedy serves as an approximate best response oracle for the influencer.

Lemma 3: *For any adversary mixed strategy $y \in \Delta^{|\mathcal{P}^*|}$, running greedy with the objective*

$$\max_S \sum_{\boldsymbol{\theta} \in \mathcal{P}^*} \frac{y_{\boldsymbol{\theta}}}{g_G(\boldsymbol{\theta})} \mathop{\mathbb{E}}_{\boldsymbol{p} \sim \boldsymbol{\theta}}[f(S, \boldsymbol{p})]$$

produces a $(1 - 1/e)$-approximate best response to y.

Proof. Since $f(\cdot, \boldsymbol{p})$ is submodular for any $\boldsymbol{p}$, the funtion $\mathbb{E}_{\boldsymbol{p} \sim \boldsymbol{\theta}}[f(\cdot, \boldsymbol{p})]$ is submodular as well, since a nonnegative linear combination of submodular functions is submodular. The final expression is another nonnegative linear combination where each term is weighted by $\frac{y_{\boldsymbol{\theta}}}{g_G(\boldsymbol{\theta})}$. Hence, greedy obtains a $1 - 1/e$ approximation to the objective, which is the same as saying that it is a $(1 - 1/e)$-approximate best response □

Lastly, we verify the data-dependent guarantee for DOSIM in the dynamic setting.

Lemma 4:‘ *If the influencer oracle achieves an α-approximation for any $\boldsymbol{\theta} \in \mathcal{P}^*$ on a specific graph G, then DOSIM provides an (α, ϵ)-minimax robust solution on G.*

Proof of Lemma 4. Provided that greedy obtains an α-approximation for any $\boldsymbol{\theta} \in \mathcal{P}^*$ for G specifically, the chain of inequalities in the proof of Theorem 2 all hold for G. □

B Additional experimental results

In this section, we provide experimental results which were deferred from the main text due to lack of space.

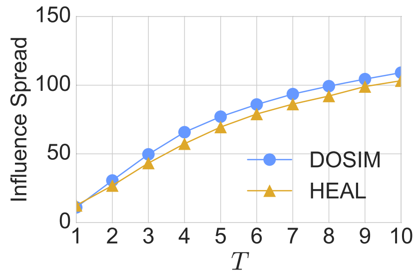

Figure 1: Influence spread as T varies on Network B.

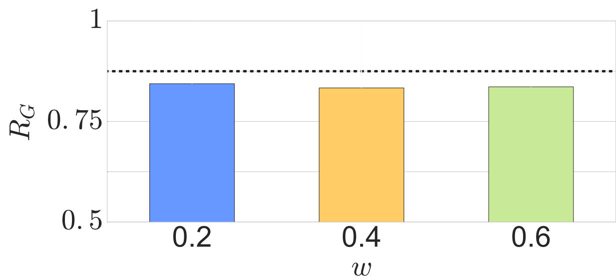

Figure 2: R_G achieved by DOSIM with half-sized intervals compared to when full intervals are known, on Network B.

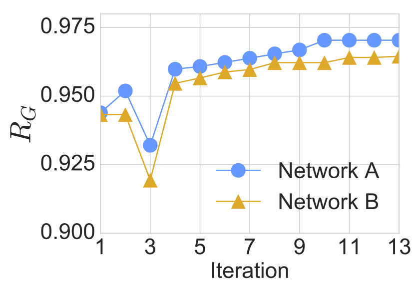

Figure 3: Convergence of DOSIM with half-sized uncertainty intervals.